WS #4-3
Exponential Functions

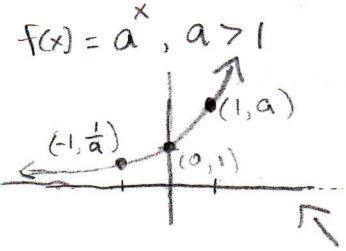
1. You will be responsible to read the section completely and review the definitions and application of the following:

A. Laws of Exponents → $a, b, m, \& n$ are real #'s w/ $a>0$ & $b>0$

1. $a^m \cdot a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^n = a^n b^n$
4. $1^n = 1$
5. $a^{-n} = 1/a^n = \left(\frac{1}{a}\right)^n$
6. $a^0 = 1$

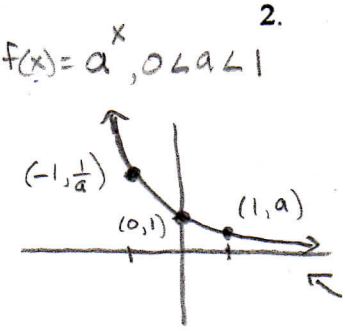
B. Exponential Functions

1. Properties of Exponential Equations $f(x) = a^x$, $a > 1$



a. $D: (-\infty, \infty)$; $R: (0, \infty)$
b. x-int → none ; y-int = 1
c. Horizontal Asymptote : x-axis ($y=0$) as $x \to -\infty$
d. Increasing ; one-to-one ; smooth ; continuous
e. Graph

2. Properties of Exponential Equations $f(x) = a^x$, $0 < a < 1$



a. $D: (-\infty, \infty)$; $R (0, \infty)$
b. x-int = none ; y-int = 1
c. Horizontal Asymptote : x-axis ($y=0$) as $x \to \infty$
d. Decreasing ; one-to-one ; smooth ; continuous
e Graph

C. Power Functions : $g(x) = x^n$, $n \geq 2$

- In a power function, the base is a variable & the exponent is a constant

(What is the difference between a power function and an exponential function?)

- In an exponential function, the base is a constant & the exponent is a variable.

D. Number e

$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e \approx 2.718$ * e is an irrational number such as π.

E. Exponential Equations

If $a^u = a^v$, then $u = v$

2. Explain the transformations of $f(x) = -e^{x-3}$ from $f(x) = e^x$

- reflect about the x-axis → $-e^x$ (mult each y-coord by -1)
- Shift right 3 units → $-e^{x-3}$ (add 3 to each x-coordinate)

3. Solve:

A. $3^{x+1} = 81$

$3^{x+1} = 3^4$

* If $a^u = a^v$, then $u = v$.

$x + 1 = 4$

$\boxed{x = 3}$

B. $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$

$e^{-x^2} = e^{2x} \cdot e^{-3}$

$-x^2 = 2x - 3$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\boxed{x = -3 \text{ or } x = 1}$

4. Between 9:00PM and 10:00PM cars arrive at Burger King's drive-thru at the rate of 12 cars per hour (.2 car per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 9:00PM.

$$F(t) = 1 - e^{-0.2t}$$

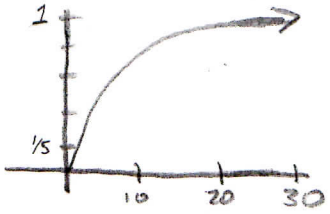
A. Determine the probability that a car will arrive within 5 minutes of 9 PM.
B. Determine the probability that a car will arrive within 30 minutes of 9 PM
C. Graph F using your calculator.
D. What value does F approach as t becomes unbounded in the positive direction?

A. Evaluate $F(t)$ at $t = 5$ mins

$F(5) = 1 - e^{-0.2(5)} \approx$ 63% probability that a car will arrive within 5 mins.

B. $F(30) = 1 - e^{-0.2(30)} \approx .9975$, so there is a 99.75% probability that a car will arrive within 30 mins.

C.



D. The value that F approaches can be found by letting $t \to \infty$. Since $e^{-0.2t} = \frac{1}{e^{0.2t}}$, it follows that $e^{-0.2t} \to 0$ as $t \to \infty$. Therefore, F approaches 1 as t gets large in $F(t) = 1 - e^{-0.2t}$

4.3 - Exponential Functions (continued Notes)

30.) F

$y = 3^{x-1}$

graph of $y = 3^x$ to the right 1.

32.) H

$y = 1 - 3^x$

graph of $y = 3^x$ reflected across x-axis + shifted up 1.

34.) C

$y = -3^x$

graph of $y = 3^x$ reflected across x-axis

36.) G

$y = 3^{1-x} \rightarrow y = 3^{-(x-1)}$

graph of $y = 3^x$ reflected across the y-axis and shifted right 1.

- Solve each equation.

54.) $5^{1-2x} = \frac{1}{5} \rightarrow 5^{1-2x} = 5^{-1} \rightarrow 1 - 2x = -1 \rightarrow \frac{-2x}{-2} = \frac{-2}{-2}$

same bases, so set exponents equal to each other + solve.

$\boxed{x = 1}$

56.) $4^{x^2} = 2^x \rightarrow [(2)^2]^{x^2} = 2^x \rightarrow 2^{2x^2} = 2^x \rightarrow 2x^2 = x$

same bases

$2x^2 - x = 0 \rightarrow x(2x-1) = 0 \rightarrow \boxed{x = 0, \frac{1}{2}}$

60.) $\left(\frac{1}{2}\right)^{1-x} = 4 \rightarrow \left(\frac{1}{2}\right)^{1-x} = \left(\frac{1}{2}\right)^{-2} \rightarrow 1 - x = -2 \rightarrow \boxed{3 = x}$

68.) If $2^x = 3$, what does 4^{-x} equal?

* How do you make 2^x become 4^{-x}? $\rightarrow (2^x)^{-2} = (3)^{-2}$

$(2^2)^{-x} = (3)^{-2}$

$4^{-x} = \boxed{\frac{1}{9}}$

70.) If $5^{-x} = 3$, what does 5^{3x} equal?

* How do you make 5^{-x} become 5^{3x}?

$(5^{-x})^{-3} = (3)^{-3}$

$5^{3x} = \boxed{\frac{1}{27}}$

72.) $y = a^x$; to find a, ask yourself "what is y when $x = 1$?"

$y = 5$, so: $\boxed{y = 5^x}$